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Technical Notes

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Comparison of Four Turbulence Models for Wall-Bounded Flows Affected by Transverse Curvature

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I. Introduction

IN many engineering applications, such as flow along ships, missiles, aircrafts, torpedoes, and underwater towed cables, and during fabrication of glass and polymer fibers, the boundary-layer thickness δ can be of the order of or greater than the body radius and thus the transverse curvature effects become important, i.e., the boundary layers are thick. The objective of the present work is to check the applicability of four eddy-viscosity-based turbulence models for two configurations of flows affected by transverse curvature: 1) the zero pressure-gradient boundary layer developing on the surface of a circular cylinder of radius a aligned with the flow and 2) the axial fully developed flow between concentric cylinders. For turbulent flow, two parameters a^+ and δ/a can be used to represent the effects of transverse curvature in the inner and the outer parts of flow, respectively; $a^+ = au_\tau/\nu$. (For the second flow type, δ denotes the radial location of zero shear stress from the inner cylinder surface of radius a.)

The four models are the low-Reynolds-number (low-Re) and two-layer k-ϵ models recently developed by Michelassi et al.[1] and Rodi et al.[2] [termed the Rodi–Mansour (RM) and two-layer (TL) models, respectively] on the basis of direct numerical simulation (DNS) data for planar wall-bounded flows, the low-Re k-ϵ model of Chien (CH), which is quite accurate for planar wall-bounded flows,[3] and the simple zero equation model of Cebeci and Smith.[4] Earlier Cebeci[4,5] had used the zero equation models to compute thick axisymmetric boundary layers, using, however, the displacement thickness as defined for planar flows to calculate the eddy viscosity in the outer region. The zero equation model in the latter paper,[5] which was supposed to be an improvement on the standard model in Ref. 4 for thick axisymmetric boundary layers, actually turns out to be worse for flows with small a^+ as compared with the now available accurate experimental data.[6,7] In Refs. 8 and 9 the near-wall region of thick axisymmetric boundary layers has been modeled using two different mixing length distributions.

In the experimental studies on thick axisymmetric boundary layers,[6,7,10] mostly mean velocity profiles have been measured and measurements of turbulent quantities, especially for small a^+ and large δ/a, are very limited. The skin-friction coefficient has been deduced from the mean velocity profile.[6,7,10] An added problem in the experiments is the difficulty in maintaining perfect axisymmetry of flow. These problems are avoided in DNS but with a penalty of not being able to achieve high Reynolds numbers.[11]

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II. Results

The two definitions of the displacement thickness, one for axisymmetric flows (δ^*_{ax}) and the other for planar flows (δ^*_{pl}), are used to compute eddy viscosity in the outer region (for definitions see Refs. 6 and 7) using the zero equation model. The corresponding models are denoted by the zero equation (δ^*_{ax}) and zero equation (δ^*_{pl}) models, respectively. With an increase in δ/a, the ratio $\delta^*_{ax}/\delta^*_{pl}$ increases and only in the limit of $\delta/a \to 0$, $\delta^*_{ax}/\delta^*_{pl} \to 1$. For the axial developed flow between cylinders the intermittency is taken equal to 1.0 throughout. The equations were solved using the control volume method. For the boundary-layer flow, 120 radial grids were taken, and for the axial developed flow between concentric cylinders, 90 radial grids were taken. To resolve sharp gradients close to the wall, about 50 grids in both the flows were located within $y^+ = 50$. For details of code validation see Ref. 12.

A. Thick Axisymmetric Boundary-Layer Flow

1. Effect of Transverse Curvature on the Predicted C_f

For this flow configuration 10 cases with different values of δ/a and a^+ were studied. A plot of C_f vs a^+ (Fig. 1) shows that with an increase in a^+ the deviation between the predicted and measured C_f decreases (hereafter when we write deviation, it means the deviation between the measurements and the predictions by the five models and the deviation between the predictions themselves). For $a^+ \simeq 30$, the zero equation (δ^*_{ax}) model underpredicts C_f by about 15%. At this a^+, the other models underpredict C_f by larger amounts, with maximum underprediction of about 45% by Chien's model. At $a^+ \simeq 300$, the values of C_f predicted by the five models are within experimental uncertainty.[6] At $a^+ \simeq 300$, however, the transverse curvature effects may still be important in the near-wall region, but for $a^+ \geq 500$ the curvature effects appear to be small. It can be shown that, in contrast, the transverse curvature effects in the outer region (represented by δ/a) have negligible effect on the predicted C_f.[12]

In Fig. 2 the predicted mean velocity profiles for $a^+ = 46.2$ are compared with the measurements.[6] To remove the dependence on C_f, whose predicted and measured values are different, the mean velocity profiles are plotted as u/u_∞ vs y/θ_{ax} (for the definition of θ_{ax} see Refs. 6 and 7), so that a direct comparison of just velocity can

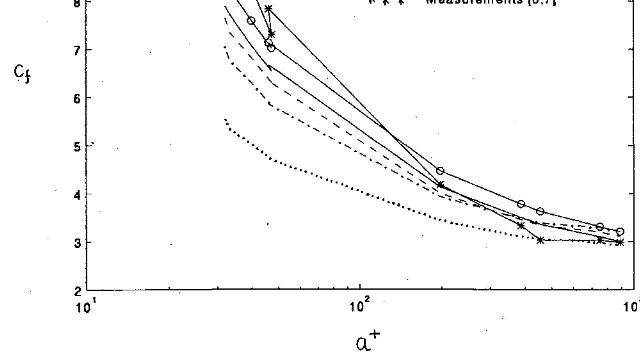

Fig. 1 Thick axisymmetric boundary-layer flow: the predicted skin-friction coefficient compared with the measurements.

Table 1 Comparison of the predicted skin-friction coefficient with the DNS data for axial fully developed flow between concentric cylinders

	Quantity	DNS[11]	Zero eq. (δ^*_{ax}) model	Zero eq. (δ^*_{pl}) model	TL model	RM model	CH model
$\delta/a = 5$ and $a^+ = 43$	C_f	0.00807	0.00867	0.00808	0.00931	0.00809	0.00605
$\delta/a = 11$ and $a^+ = 21$	C_f	0.00987	0.01123	0.01030	0.01106	0.00939	0.00689

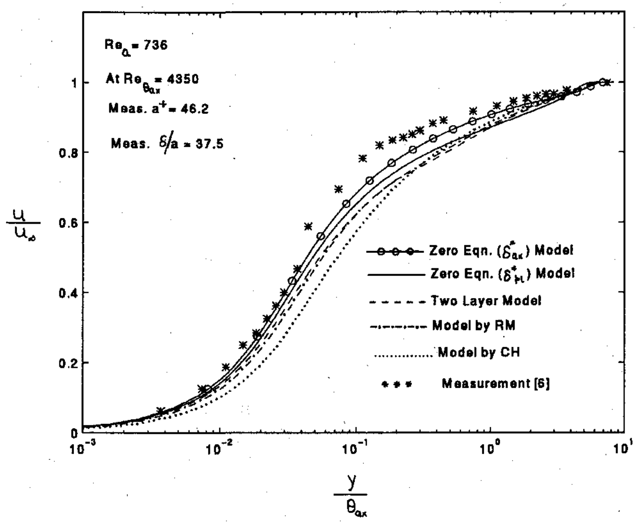

Fig. 2 Thick axisymmetric boundary-layer flow: the predicted mean velocity profiles compared with the measurements.

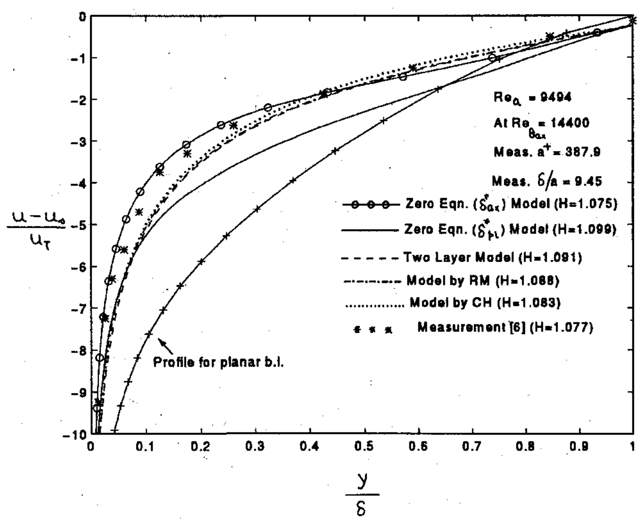

Fig. 3 Thick axisymmetric boundary-layer flow: the predicted mean velocity-defect profiles compared with the measurements; the shape factor $H = \delta^*_{ax}/\theta_{ax}$.

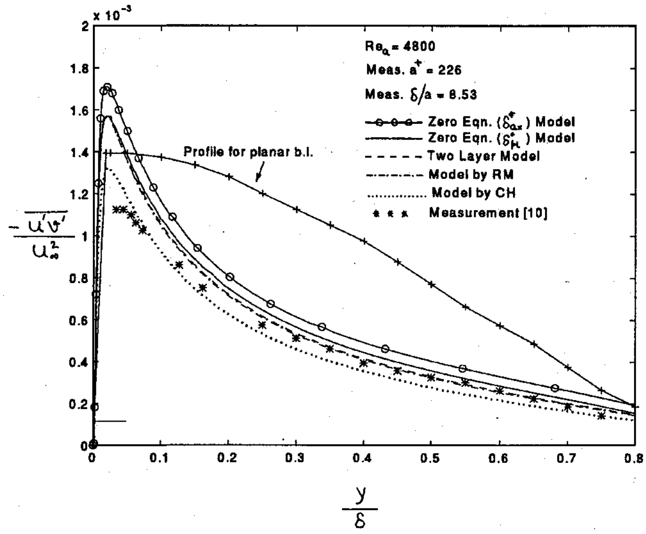

Fig. 4 Thick axisymmetric boundary-layer flow: the predicted Reynolds shear stress profiles compared with the measurements.

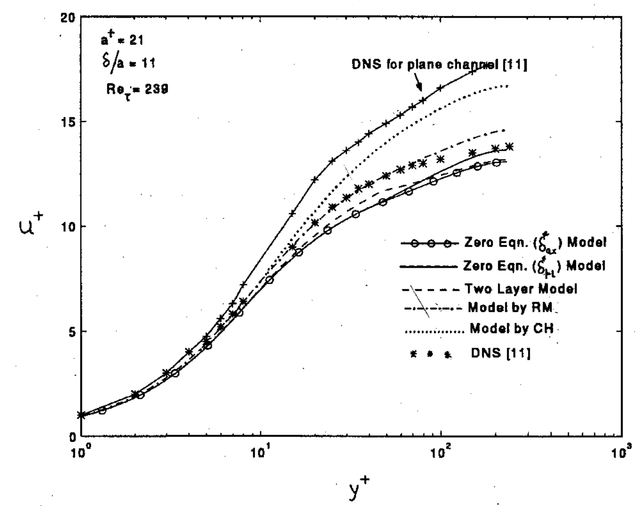

Fig. 5 Axial developed flow between concentric cylinders: the predicted mean velocity profiles compared with the direct numerical simulation data.

be made. The predicted mean velocity profiles show large deviation in the near-wall region, consistent with the deviations found in the C_f values. The use of the standard wall variables u^+ and y^+, which contain the dependence on C_f through the friction velocity, would show the maximum deviation at the edge of the boundary layer.

2. Flow in the Outer Region

Willmarth et al.[6] showed that as δ/a increases, the velocity profile in the outer region becomes flatter as compared with that on a flat plate. In the 10 cases studied here, the measured values of the shape factor, $H = \delta^*_{ax}/\theta_{ax}$, vary from about 1.01 to about 1.2, in contrast to $H \simeq 1.4$ for the planar turbulent boundary layer. All of the models also predict the flatter velocity profiles for thick axisymmetric boundary layers: the computed values of H for the 10 cases from all of the 5 models are within 10% of the measured values (not shown here). In Fig. 3, the computed mean velocity-defect profiles for one typical case at $a^+ = 387.9$ and $\delta/a = 9.45$ are compared with the measurements.[6] The best agreement between the predictions and the measurements is obtained by the zero equation (δ^*_{ax}) model; note that, in contrast, the zero equation (δ_{pl}) model does not perform well. Away from the wall ($y/\delta \geq 0.2$) predictions by the k-ϵ models are also reasonably accurate. Even for the smaller a^+ values, for which all of the models underpredict C_f, the predictions of the velocity profiles in the outer coordinates (and with the dependence on C_f removed by taking same u_τ) follow the same trends as in Fig. 3.

From Fig. 4, it is clear that all of the models are reasonably accurate in predicting a rapid decay of the Reynolds shear stress in the outer region of thick axisymmetric boundary layers as compared with that for the flat plate boundary layer. Close to the wall, there are large differences between the predictions by the models and the measurement. We believe this is caused by an error in the measurement. An estimate of the Reynolds stress variation very close to the wall from the constant shear moment condition shows that it is significantly higher than the measurements and closer to the predictions by the models. The profiles of turbulent kinetic energy also follow the same trends as the Reynolds shear stress (not shown here).

B. Axial Developed Flow Between Concentric Cylinders

For this flow type, two cases were studied (the predicted values of C_f are shown in Table 1). For one typical flow at $a^+ = 21$ and $\delta/a = 11$, even close to the wall all of the models predict deviation in the mean velocity profile from that for the plane channel, in agreement with the DNS data (Fig. 5). The complete velocity profile computed by the low-*Re* k-ϵ RM model is in the best agreement with the DNS data. The Reynolds stress predicted by all of the models, except Chien's model, is also in very good agreement (almost coincident) with the DNS data. The four models (except Chien's model) accurately predict significant reduction in the Reynolds stress in the wall variables as compared with that for the plane channel (not shown here).

III. Discussion and Concluding Remarks

From the results presented in Sec. II it is clear that for the axial developed flow between concentric cylinders most models (except Chien's model) predict all quantities in reasonably good agreement with the DNS data. In contrast, for thick axisymmetric boundary layers at small a^+, the five models underpredict turbulence quantities and C_f by large amount.

To understand the difference in the predictions for the two flow configurations we need to consider the details of the flows. One difference between the two flow configurations is in the Reynolds number. Thick axisymmetric boundary layers are at a much higher Reynolds number as compared with that for the axial developed flow between concentric cylinders. Another possibility is that the measurements for the boundary-layer flow are in error because of the misalignment of the cylinder and/or because C_f is incorrectly deduced. Recently, Bull and Dekkars[13] have shown that in the boundary-layer flow at small radius Reynolds numbers (Re_a), even at about 1-deg yaw angle, vortex shedding from the cylinder may lead to additional turbulence generation. A third possibility is that the flow structure in the two cases is different. In the experiments on thick axisymmetric boundary layers,[7] large eddies scaling with the boundary-layer thickness have been observed to move across the cylinder. This is not accounted for in any of the models. This viscous-inviscid interaction is absent in DNS of axial developed flow between concentric cylinders.[11]

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Performance of Eddy-Viscosity-Based Turbulence Models in Three-Dimensional Turbulent Interaction

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I. Introduction

In recent years, the numerical simulation of three-dimensional turbulent interactions has received considerable impetus from developments in two different areas: Riemann solvers and turbulence modeling. Sophisticated upwinded methods for the inviscid terms of the Navier–Stokes equations obviate the need for specification of arbitrary damping coefficients. Solvers employing such high-resolution flux evaluation techiques are superior to central-difference-based schemes not only in shock capturing but also in resolving smoother viscous-dominated regions of the flow.[1]

Advances in eddy-viscosity-based turbulence modeling have been reported principally in one- and two-equation formulations.[2–4] Although turbulence model development and evaluation efforts have focused on two-dimensional interactions, most practical flows are inherently three-dimensional. Fewer efforts have systematically investigated the performance of popular turbulence models in three-dimensional flows, where the physical processes are significantly different from those found in two dimensions.[5] This work examines several eddy-viscosity turbulence models for their ability to reproduce the experimental data in a strong three-dimensional external-shock-wave–turbulent-boundary-layer interaction. The four models investigated are the zero-equation Baldwin–Lomax (BL) model,[6] the one-equation models of Baldwin and Barth[2] (BB) and of Spalart and Allmaras[3] (SA), and the two-equation k–ϵ closure.[7]

The quantitative evaluation of turbulence models in three dimensions has been significantly aided by recent measurements not only of surface pressure but also of gradient quantities closely linked to the viscous and turbulent stresses. The error bounds on such data are typically less than 10%. The experimental geometries are simple, typically a combination of fins on a plate or cylinder–flare intersections. For fin-based configurations, computational results with zero- and two-equation models[8] indicate that good agreement is achieved with experimental data for surface pressure, perhaps as a consequence of the inviscid rotational nature of the dominant portion of the flowfield. However, the effort to match derivative quantities such as skin friction coefficients and heat transfer rates has proven to be considerably more challenging (see Ref. 9 for example).

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